

Appendix 1 NATIONAL IMAGE INTERPRETABILITY RATING SCALES

The aerial imaging community utilizes the National Imagery Interpretability Rating Scale (NIIRS) to define and measure the quality of images and performance of imaging systems. Through a process referred to as "rating" an image, the NIIRS is used by imagery analysts to assign a number that indicates the interpretability of a given image. The NIIRS concept provides a means to directly relate the quality of an image to the interpretation tasks for which it may be used. Although the NIIRS has been primarily applied in the evaluation of aerial imagery, it provides a systematic approach to measuring the quality of photographic or digital imagery, the performance of image capture devices, and the effects of image processing algorithms.

<http://www.fas.org/irp/imint/niirs.htm>

references:

Civil NIIRS Reference Guide, http://www.fas.org/irp/imint/niirs_c/index.html,

From the Imagery Resolution Assessments and Reporting Standards (IRARS) Committee.

Imagery Interpretability Rating Scales,

L. A. Maver, C. D. Erdman, K. Riehl, Itek Optical Systems, Lexington, MA, Society for Information Display, 1995, Applied Vision/Human Factors session,

<http://www.sid.org/sid95pdf/10-2x.pdf>

This paper describes the NIIRS and provides information on how the scale is developed and used to facilitate its application to other imaging scenarios.

1 NIIRS 0

Visible NIIRS	Radar NIIRS	Infrared NIIRS	Multispectral NIIRS
March 1994	August 1992	April 1996	February 1995
Interpretability of the imagery is precluded by obscuration, degradation, or very poor resolution	Interpretability of the imagery is precluded by obscuration, degradation, or very poor resolution	Interpretability of the imagery is precluded by obscuration, degradation, or very poor resolution	Interpretability of the imagery is precluded by obscuration, degradation, or very poor resolution

2 NIIRS 1 [over 9.0 m GRD]

Visible NIIRS	Radar NIIRS	Infrared NIIRS	Multispectral NIIRS
Detect a medium-sized port facility and/or distinguish between taxiways and runways at a large airfield.	Detect the presence of aircraft dispersal parking areas. Detect a large cleared swath in a densely wooded area. Detect, based on presence of piers and warehouses, a port facility. Detect lines of transportation (either road or rail), but do not distinguish between	Distinguish between runways and taxiways on the basis of size, configuration or pattern at a large airfield. Detect a large (e.g., greater than 1 square kilometer) cleared area in dense forest. Detect large ocean-going vessels (e.g., aircraft carrier, super-tanker, KIROV) in open water. Detect large areas (e.g., greater than 1 square kilometer) of marsh/swamp.	Distinguish between urban and rural areas. Identify a large wetland (greater than 100 acres). Detect meander flood plains (characterized by features such as channel scars, oxbow lakes, meander scrolls). Delineate coastal shoreline. Detect major highway and rail bridges over water (e.g., Golden Gate, Chesapeake Bay). Delineate extent of snow or ice cover.

3 NIIRS 2 [4.5 - 9.0 m GRD]

Visible NIIRS	Radar NIIRS	Infrared NIIRS	Multispectral NIIRS
<p>Detect large hangars at airfields.</p> <p>Detect large static radars (e.g., AN/FPS-85, COBRA DANE, PECHORA, HENHOUSE).</p> <p>Detect military training areas.</p> <p>Identify an SA-5 site based on road pattern and overall site configuration.</p> <p>Detect large buildings at a naval facility (e.g., warehouses, construction hall).</p> <p>Detect large buildings (e.g., hospitals, factories).</p>	<p>Detect the presence of large (e.g., BLACKJACK, CAMBER, COCK, 707, 747) bombers or transports.</p> <p>Identify large phased array radars (e.g., HEN HOUSE, DOG HOUSE) by type.</p> <p>Detect a military installation by building pattern and site configuration.</p> <p>Detect road pattern, fence, and hardstand configuration at SSM launch sites (missile silos, launch control silos) within a known ICBM complex.</p> <p>Detect large non-combatant ships (e.g., freighters or tankers) at a known port facility.</p> <p>Identify athletic stadiums.</p>	<p>Detect large aircraft (e.g., C-141, 707, BEAR, CANDID, CLASSIC).</p> <p>Detect individual large buildings (e.g., hospitals, factories) in an urban area.</p> <p>Distinguish between densely wooded, sparsely wooded and open fields.</p> <p>Identify an SS-25 base by the pattern of buildings and roads.</p> <p>Distinguish between naval and commercial port facilities based on type and configuration of large functional areas.</p>	<p>Detect multilane highways.</p> <p>Detect strip mining.</p> <p>Determine water current direction as indicated by color differences (e.g., tributary entering larger water feature, chlorophyll or sediment patterns).</p> <p>Detect timber clear-cutting.</p> <p>Delineate extent of cultivated land.</p> <p>Identify riverine flood plains.</p>

4 NIIRS 3 [2.5 - 4.5 m GRD]

Visible NIIRS	Radar NIIRS	Infrared NIIRS	Multispectral NIIRS
<p>Identify the wing configuration (e.g., straight, swept, delta) of all large aircraft (e.g., 707, CONCORD, BEAR, BLACKJACK).</p> <p>Identify radar and guidance areas at a SAM site by the configuration, mounds, and presence of concrete aprons.</p> <p>Detect a helipad by the configuration and markings.</p> <p>Detect the presence / absence of support vehicles at a mobile missile base.</p> <p>Identify a large surface ship in port by type (e.g., cruiser, auxiliary ship, noncombatant/merchant).</p> <p>Detect trains or strings of standard rolling stock on railroad tracks (not individual cars)</p>	<p>Detect medium-sized aircraft (e.g., FENCER, FLANKER, CURL, COKE, F-15).</p> <p>Identify an ORBITA site on the basis of a 12 meter dish antenna normally mounted on a circular building.</p> <p>Detect vehicle revetments at a ground forces facility.</p> <p>Detect vehicles/pieces of equipment at a SAM, SSM, or ABM fixed missile site.</p> <p>Determine the location of the superstructure (e.g., fore, amidships, aft) on a medium-sized freighter.</p> <p>Identify a medium-sized (approx. six track) railroad classification yard.</p>	<p>Distinguish between large (e.g., C-141, 707, BEAR, A300 AIRBUS) and small aircraft (e.g., A-4, FISHBED, L-39).</p> <p>Identify individual thermally active flues running between the boiler hall and smoke stacks at a thermal power plant.</p> <p>Detect a large air warning radar site based on the presence of mounds, revetments and security fencing.</p> <p>Detect a driver training track at a ground forces garrison.</p> <p>Identify individual functional areas (e.g., launch sites, electronics area, support area, missile handling area) of an SA-5 launch complex.</p> <p>Distinguish between large (e.g, greater than 200 meter) freighters and tankers.</p>	<p>Detect vegetation/soil moisture differences along a linear feature (suggesting the presence of a fenceline).</p> <p>Identify major street patterns in urban areas.</p> <p>Identify golf courses.</p> <p>Identify shoreline indications of predominant water currents.</p> <p>Distinguish among residential, commercial, and industrial areas within an urban area.</p> <p>Detect reservoir depletion.</p>

5 NIIRS 4 [1.2 - 2.5 m GRD]

Visible NIIRS	Radar NIIRS	Infrared NIIRS	Multispectral NIIRS
<p>Identify all large fighters by type (e.g., FENCER, FOXBAT, F-15, F-14). Detect the presence of large individual radar antennas (e.g., TALL KING). Identify, by general type, tracked vehicles, field artillery, large river crossing equipment, wheeled vehicles when in groups. Detect an open missile silo door. Determine the shape of the bow (pointed or blunt/rounded) on a medium-sized submarine (e.g., ROMEO, HAN, Type 209, CHARLIE 11, ECHO 11, VICTOR II/III). Identify individual tracks, rail pairs, control towers,</p>	<p>Distinguish between large rotary-wing and medium fixed-wing aircraft (e.g., HALO helicopter versus CRUSTY transport). Detect recent cable scars between facilities or command posts. Detect individual vehicles in a row at a known motor pool. Distinguish between open and closed sliding roof areas on a single bay garage at a mobile missile base. Identify square bow shape of ROPUCHA class (LST). Detect all rail/road bridges.</p>	<p>Identify the wing configuration of small fighter aircraft (e.g., FROGFOOT, F- 16, FISHBED). Detect a small (e.g., 50 meter square) electrical transformer yard in an urban area. Detect large (e.g., greater than 10 meter diameter) environmental domes at an electronics facility. Detect individual thermally active vehicles in garrison. Detect thermally active SS-25 MSV's in garrison. Identify individual closed cargo hold hatches on large merchant ships.</p>	<p>Detect recently constructed weapon positions (e.g. tank, artillery, self-propelled gun) based on the presence of revetments, berms, and ground scarring in vegetated areas. Distinguish between two-lane improved and unimproved roads. Detect indications of natural surface airstrip maintenance or improvements (e.g., runway extension, grading, resurfacing, bush removal, vegetation cutting). Detect landslide or rockslide large enough to obstruct a single-lane road. Detect small boats(15-20 feet in length) in open water</p>

6 NIIRS 5 [0.75 - 1.2 m GRD]

Visible NIIRS	Radar NIIRS	Infrared NIIRS	Multispectral NIIRS
<p>Distinguish between a MIDAS and a CANDID by the presence of refueling equipment (e.g., pedestal and wing pod).</p> <p>Identify radar as vehicle-mounted or trailer-mounted.</p> <p>Identify, by type, deployed tactical SSM systems (e.g., FROG, SS-21, SCUD).</p> <p>Distinguish between SS-25 mobile missile TEL and Missile Support Vans (MSVS) in a known support base, when not covered by camouflage.</p> <p>Identify TOP STEER or TOP SAIL air surveillance radar on KIROV-, SOVREMENNY-, KIEV-, SLAVA-, MOSKVA-, KARA-, or KRESTA-II-class vessels.</p>	<p>Count all medium helicopters (e.g., HIND, HIP, HAZE, HOUND, PUMA, WASP).</p> <p>Detect deployed TWIN EAR antenna.</p> <p>Distinguish between river crossing equipment and medium/heavy armored vehicles by size and shape (e.g., MTU-20 vs. T-62 MBT).</p> <p>Detect missile support equipment at an SS-25 RTP (e.g., TEL, MSV).</p> <p>Distinguish bow shape and length/width differences of SSNS.</p> <p>Detect the break between railcars (count railcars).</p>	<p>Distinguish between single-tail (e.g., FLOGGER, F-16, TORNADO) and twin-tailed (e.g., F-15, FLANKER, FOXBAT) fighters.</p> <p>Identify outdoor tennis courts.</p> <p>Identify the metal lattice structure of large (e.g. approximately 75 meter) radio relay towers.</p> <p>Detect armored vehicles in a revetment.</p> <p>Detect a deployed TET (transportable electronics tower) at an SA-10 site.</p> <p>Identify the stack shape (e.g., square, round, oval) on large (e.g., greater than 200 meter) merchant ships.</p>	<p>Detect automobile in a parking lot.</p> <p>Identify beach terrain suitable for amphibious landing operation.</p> <p>Detect ditch irrigation of beet fields.</p> <p>Detect disruptive or deceptive use of paints or coatings on buildings/structures at a ground forces installation.</p> <p>Detect raw construction materials in ground forces deployment areas (e.g., timber, sand, gravel).</p>

7 NIIRS 6 [0.40 - 0.75 m GRD]

Visible NIIRS	Radar NIIRS	Infrared NIIRS	Multispectral NIIRS
<p>Distinguish between models of small/medium helicopters (e.g., HELIX A from HELIX B from HELIX C, HIND D from HIND E, HAZE A from HAZE B from HAZE C). Identify the shape of antennas on EW/GCI/ACQ radars as parabolic, parabolic with clipped comers or rectangular. Identify the spare tire on a medium-sized truck. Distinguish between SA-6, SA-11, and SA-17 missile airframes. Identify individual launcher covers (8) of vertically launched SA-N-6 on SLAVA-class vessels. Identify automobiles as</p>	<p>Distinguish between variable and fixed-wing fighter aircraft (e.g., FENCER vs. FLANKER). Distinguish between the BAR LOCK and SIDE NET antennas at a BAR LOCK/SIDE NET acquisition radar site. Distinguish between small support vehicles (e.g., UAZ-69, UAZ-469) and tanks (e.g., T-72, T-80). Identify SS-24 launch triplet at a known location. Distinguish between the raised helicopter deck on a KRESTA II (CG) and the helicopter deck with main deck on a KRESTA I (CG).</p>	<p>Detect wing-mounted stores (i.e., ASM, bombs) protruding from the wings of large bombers (e.g., B-52, BEAR, Badger). Identify individual thermally active engine vents atop diesel locomotives. Distinguish between a FIX FOUR and FIX SIX site based on antenna pattern and spacing. Distinguish between thermally active tanks and APCs. Distinguish between a 2-rail and 4-rail SA-3 launcher. Identify missile tube hatches on submarines.</p>	<p>Detect summer woodland camouflage netting large enough to cover a tank against a scattered tree background. Detect foot trail through tall grass. Detect navigational channel markers and mooring buoys in water. Detect livestock in open but fenced areas. Detect recently installed minefields in ground forces deployment area based on a regular pattern of disturbed earth or vegetation. Count individual dwellings in subsistence housing areas (e.g., squatter settlements, refugee camps).</p>

8 NIIRS 7 [0.20 - 0.40 m GRD]

Visible NIIRS	Radar NIIRS	Infrared NIIRS	Multispectral NIIRS
<p>Identify fitments and fairings on a fighter-sized aircraft (e.g., FULCRUM, FOXHOUND).</p> <p>Identify ports, ladders, vents on electronics vans.</p> <p>Detect the mount for antitank guided missiles (e.g., SAGGER on BMP-1).</p> <p>Detect details of the silo door hinging mechanism on Type III-F, III-G, and 11-H launch silos and Type III-X launch control silos.</p> <p>Identify the individual tubes of the RBU on KIROV-, KARA-, KRIVAK-class vessels.</p> <p>Identify individual rail ties.</p>	<p>Identify small fighter aircraft by type (e.g., FISHBED, FITTER, FLOGGER).</p> <p>Distinguish between electronics van trailers (without tractor) and van trucks in garrison.</p> <p>Distinguish, by size and configuration, between a turreted, tracked APC and a medium tank (e.g., BMP-1/2 vs. T-64).</p> <p>Detect a missile on the launcher in an SA-2 launch revetment.</p> <p>Distinguish between bow mounted missile system on KRIVAK I/II and bow mounted gun turret on KRIVAK III.</p> <p>Detect road/street lamps in an urban residential area or military complex.</p>	<p>Distinguish between ground attack and interceptor versions of the MIG-23 FLOGGER based on the shape of the nose.</p> <p>Identify automobiles as sedans or station wagons.</p> <p>Identify antenna dishes (less than 3 meters in diameter) on a radio relay tower.</p> <p>Identify the missile transfer crane on a SA-6 transloader.</p> <p>Distinguish between an SA-2/CSA-1 and a SCUD-B missile transporter when missiles are not loaded.</p> <p>Detect mooring cleats or bollards on piers.</p>	<p>Distinguish between tanks and three-dimensional tank decoys.</p> <p>Identify individual 55-gallon drums.</p> <p>Detect small marine mammals (e.g., harbor seals) on sand/gravel beaches.</p> <p>Detect underwater pier footings.</p> <p>Detect foxholes by ring of spoil outlining hole.</p> <p>Distinguish individual rows of truck crops.</p>

9 NIIRS 8 [0.10 - 0.20 m GRD]

Visible NIIRS	Radar NIIRS	Infrared NIIRS	Multispectral NIIRS
<p>Identify the rivet lines on bomber aircraft.</p> <p>Detect horn-shaped and W-shaped antennas mounted atop BACKTRAP and BACKNET radars.</p> <p>Identify a hand-held SAM (e.g., SA-7/14, REDEYE, STINGER).</p> <p>Identify joints and welds on a TEL or TELAR.</p> <p>Detect winch cables on deck-mounted cranes.</p> <p>Identify windshield wipers on a vehicle.</p>	<p>Distinguish the fuselage difference between a HIND and a HIP helicopter.</p> <p>Distinguish between the FAN SONG E missile control radar and the FAN SONG F based on the number of parabolic dish antennas (three vs. one).</p> <p>Identify the SA-6 transloader when other SA-6 equipment is present.</p> <p>Distinguish limber hole shape and configuration differences between DELTA I and YANKEE I (SSBNs).</p> <p>Identify the dome/vent pattern on rail tank cars.</p>	<p>Identify the RAM airscoop on the dorsal spine of FISHBED J/K/L.</p> <p>Identify limbs (e.g., arms, legs) on an individual.</p> <p>Identify individual horizontal and vertical ribs on a radar antenna.</p> <p>Detect closed hatches on a tank turret.</p> <p>Distinguish between fuel and oxidizer Multi-System Propellant Transporters based on twin or single fitments on the front of the semi-trailer.</p> <p>Identify individual posts and rails on deck edge life rails.</p>	

10 NIIRS 9 [less than 0.10 m GRD]

Visible NIIRS	Radar NIIRS	Infrared NIIRS	Multispectral NIIRS
<p>Differentiate cross-slot from single slot heads on aircraft skin panel fasteners.</p> <p>Identify small light-toned ceramic insulators that connect wires of an antenna canopy.</p> <p>Identify vehicle registration numbers (VRN) on trucks.</p> <p>Identify screws and bolts on missile components.</p> <p>Identify braid of ropes (1 to 3 inches in diameter).</p> <p>Detect individual spikes in railroad ties.</p>	<p>Detect major modifications to large aircraft (e.g., fairings, pods, winglets).</p> <p>Identify the shape of antennas on EW/GCI/ACQ radars as parabolic, parabolic with clipped corners, or rectangular.</p> <p>Identify, based on presence or absence of turret, size of gun tube, and chassis configuration, wheeled or tracked APCs by type (e.g., BTR-80, BMP- 1/2, MT-LB, MI 13).</p> <p>Identify the forward fins on an SA-3 missile.</p> <p>Identify individual hatch covers of vertically launched SA-N-6 surface-to-air system.</p> <p>Identify trucks as cab-over-engine or engine-in-front.</p>	<p>Identify access panels on fighter aircraft.</p> <p>Identify cargo (e.g., shovels, rakes, ladders) in an open-bed, light-duty truck.</p> <p>Distinguish between BIRDS EYE and BELL LACE antennas based on the presence or absence of small dipole elements.</p> <p>Identify turret hatch hinges on armored vehicles.</p> <p>Identify individual command guidance strip antennas on an SA-2/CSA-1 missile.</p> <p>Identify individual rungs on bulkhead mounted ladders.</p>	

Appendix 2 DERIVATION OF THE BOHR ATOM

The existence of line spectra can be explained by means of the first 'quantum' model of the atom, developed by Bohr in 1913. Although the Bohr model of the hydrogen atom was eventually replaced, it yields the correct values for the observed spectral lines, and gives a substantial insight into the structure of atoms in general. The following derivation has the objective of obtaining the energy levels of the Bohr atom.

It is an experimental fact that the force F between two point charges q_1 and q_2 separated by a distance r is given by:

$$F = \frac{q_1 q_2}{4\pi \epsilon_0 r^2} \quad (\text{Eqn. A4-1})$$

where $\frac{1}{4\pi \epsilon_0} = 8.99 \times 10^9 \left[\frac{\text{N m}^2}{\text{C}^2} \right]$ and q_1 and q_2 are in units of Coulombs. The distance, r , is in meters of course. The potential energy associated with these two charges is:

$$U = \frac{q_1 q_2}{4\pi \epsilon_0 r} \quad (\text{Eqn. A4-2})$$

taking $U(r = \infty) = 0$. Note that the charges may be positive or negative.

For a single electron atom, we take the charge of the nucleus, q_1 , to be $+Ze$, where Z is the atomic number of the atom (the number of protons in the nucleus). Z equals 1 for hydrogen. The charge of the electron, q_2 , is $-e$. Substituting these values into (A1-1) and (A1-2) above we obtain:

$$F = - \frac{Ze^2}{4\pi \epsilon_0 r^2} \quad (\text{Eqn. A4-3})$$

and,

$$U = - \frac{Ze^2}{4\pi \epsilon_0 r} \quad (\text{Eqn. A4-4})$$

The minus sign on the force term means the force is 'inward', or attractive. A negative potential energy means the electron is in a potential 'well'. Given this expression for the potential energy, we need a similar expression for kinetic energy.

Let us assume that the electron moves in a circular orbit around the nucleus. Then Newton's second Law ($F=ma$) (here: setting the Coulomb force equal to the centripetal force) can be written as:

$$\frac{-Ze^2}{4\pi \epsilon_0 r^2} = -mv^2 / r \quad (\text{Eqn. A4-5})$$

The kinetic energy, T , is then easily obtained:

$$T = \frac{1}{2} mv^2 = \frac{1}{2} \frac{Ze^2}{4\pi \epsilon_0 r} \quad (\text{Eqn. A4-6})$$

The total energy of the electron, E, therefore is obtained:

$$E = U + T = -\frac{Ze^2}{4\pi \epsilon_0 r} + \frac{1}{2} \frac{Ze^2}{4\pi \epsilon_0 r} = -\frac{1}{2} \frac{Ze^2}{4\pi \epsilon_0 r} \quad (\text{Eqn. A4-7})$$

The total energy is negative - a general characteristic of bound orbits. This equation also tells us that if we know the radius of the orbit (r) we can calculate the energy E of the electron. Bohr now introduced the first of his two postulates, namely that the only allowed orbits were those for which the angular momentum, L, was given by:

$$L = mvr = n\hbar \quad (\text{Eqn. A4-8})$$

where:

m = electron mass

v = velocity

r = radius of the orbit

n = an integer (1,2,3,...)

and

$$\hbar = \frac{h}{2\pi} = 1.054 \times 10^{-34} \text{ [Joule-seconds]}$$

$$= 0.658 \times 10^{-15} \text{ [eV-seconds]}$$

where "h" is simply Planck's constant, as before. This is sufficient to give us:

$$v_n = \frac{n\hbar}{mr_n} \quad (\text{Eqn. A4-9})$$

for the velocity of the electron in its orbit. Note that there is an index n, for the different allowed orbits. It follows that:

$$\frac{mv_n^2}{r_n} = \frac{Ze^2}{4\pi \epsilon_0 r_n^2} = \frac{mn^2 \hbar^2}{m^2 r_n^3} \quad (\text{Eqn. A4-10})$$

Upon solving for the radius of the orbit (r_n), we get:

$$r_n = \frac{n^2 \hbar^2}{m} \times \frac{4\pi \epsilon_0}{Ze^2} = n^2 \left[\frac{4\pi \epsilon_0 \hbar^2}{Zme^2} \right] \quad (\text{Eqn. A4-11})$$

$$r_n \text{ [meters]} = n^2 \times 0.528 \times 10^{-10} / Z.$$

This only works for one electron atoms (H and He^+ as a practical matter), but within that restriction, it works fairly well. For hydrogen ($Z=1$) we get the Bohr radius, $r_1 = 0.528 \times 10^{-10}$ meters as the radius of the smallest orbit.

Substituting the expression for r_n into Eqn. A4-7, we obtain for the energy:

$$E = -\frac{1}{2} \frac{Ze^2}{4\pi\epsilon_0} \times \frac{1}{n^2} \frac{Zme^2}{4\pi\epsilon_0\hbar^2}$$

or

$$E = -\frac{1}{2} \left(\frac{Ze^2}{4\pi\epsilon_0\hbar} \right)^2 \frac{m}{n^2} = Z^2 \frac{E_1}{n^2} \quad (\text{Eqn. A4-12})$$

where

$$E_1 = - \frac{me^4}{32\pi^2\epsilon_0^2\hbar^2} = -13.58 \text{ eV}$$

is the energy of the electron in its lowest or "ground" state in the hydrogen atom.

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Appendix 3 USEFUL EQUATIONS

EM Waves

$$\lambda f = c; E = hf; \lambda = \frac{hc}{\Delta E}; c = 2.998 \times 10^8; 1 \text{ eV} = 1.602 \times 10^{-19} \text{ Joules}$$

$$h = \text{Planck's Constant} = \begin{cases} 6.626 \times 10^{-34} \text{ Joule} \cdot \text{seconds} \\ 4.136 \times 10^{-15} \text{ eV} \cdot \text{seconds} \end{cases}$$

$$\Delta E(\text{eV}) = \frac{1.24 \times 10^{-6}}{\lambda(\text{m})} = \frac{1.24}{\lambda(\text{nm})}$$

Bohr Atom:

$$r_n(\text{meters}) = n^2 \times 0.528 \times 10^{-10} / Z.$$

$$E_n = -\frac{1}{2} \left(\frac{Z e^2}{4\pi \epsilon_0 \hbar} \right)^2 \frac{m}{n^2} = Z^2 \frac{E_1}{n^2}; \quad E_1 = -\frac{me^4}{32\pi^2 \epsilon_0^2 \hbar^2} = -13.58 \text{ eV};$$

$$number \propto e^{-\frac{\text{Bandgap Energy}}{\text{Thermal Energy (kT)}}}$$

Black Body Radiation

$$c = 3 \times 10^8 \frac{\text{m}}{\text{s}}; h = 6.626 \times 10^{-34} \text{ joule} \cdot \text{s}; k = 1.38 \times 10^{-23} \frac{\text{Joule}}{\text{Kelvin}}$$

$$Radiance = L = \frac{2 hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

$$\text{Stefan Boltzmann Law: } R = \sigma T^4 \left(\frac{\text{Watts}}{\text{m}^2} \right)$$

$$\epsilon = \text{Emissivity}; \sigma = 5.67 \times 10^{-8} \left(\frac{\text{W}}{\text{m}^2 \text{K}^4} \right); T = \text{Temperature (K)}$$

$$\text{Wien's Law: } \lambda_{\text{max}} = \frac{a}{T} \quad a = 2.898 \times 10^{-3} \text{ m K}$$

Optics

$$\frac{1}{f} = \frac{1}{i} + \frac{1}{o}; \quad f/\# = \frac{\text{Focal Length}}{\text{Diameter}}$$

$$\text{Rayleigh Criteria: } \text{GSD} = \Delta q \bullet \text{altitude} = \frac{l}{\text{diameter}} \bullet \text{altitude}$$

Orbital Mechanics

$$\vec{F} = -G \frac{m_1 m_2}{r^2} \hat{r}; \quad F = g_o m \left\| \frac{R_{Earth}}{r} \right\|^2 \quad G = 6.67 \times 10^{-11} \text{ N} \frac{\text{m}^2}{\text{kg}^2}; \quad g_o = G \frac{m_{Earth}}{R_{Earth}^2} = 9.8 \frac{\text{m}}{\text{s}^2}$$

$$R_{Earth} = 6.38 \times 10^6 \text{ m}, \quad m_{Earth} = 5.9736 \times 10^{24} \text{ kg}.$$

$$v = \omega r; \quad \omega = 2\pi f; \quad t = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$F_{centripetal} = m \frac{v^2}{r} = m \omega^2 r$$

$$\text{circular motion: } v = \sqrt{\frac{g_o}{r}} R_{Earth}$$

$$\text{Ellipses: } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; \quad e = \frac{\sqrt{a^2 - b^2}}{a} \quad \text{or} \quad e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$\text{Distance from center to focus is } c = e a = \sqrt{a^2 - b^2}$$

$$\text{Elliptical orbit: } v = \sqrt{GM \left[\frac{2}{r} - \frac{1}{a} \right]}$$

$$t^2 = \frac{4\pi^2}{g_o R_{Earth}^2} r^3 = \frac{4\pi^2}{M_{Earth} G} r^3$$

Radar Resolution:

$$\text{Rayleigh Criteria: } \text{GSD} = \Delta q \bullet \text{altitude} = \frac{l}{\text{diameter}} \bullet \text{altitude}$$

$$\text{Range Resolution: Scan Mode Sar: } \text{GSD} = \frac{L}{2}$$

